$2dl/\Delta pb^2$; $\beta_2 = 2sl/\Delta pb^3$; $Sen = \tau_0[2(b - a)/\eta W]^{n/m}$, St. Venant number; $Re = \rho W^2[2(b - a)/\eta W]^{n/m}$, Reynolds number; $Ro = W/\Omega_0 a$, Rossby number.

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UNSTEADY TWO-DIMENSIONAL FLOW OF A COMPRESSIBLE NON-NEWTONIAN FLUID IN A LONG ANNULAR CHANNEL CAUSED BY THE MOTION OF AN INSIDE PIPE

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We solve the unsteady, two-dimensional problem of the hydrodynamics of a compressible non-Newtonian fluid connected with the study of the flow in an annular channel caused by the motion of an inside pipe.

One of the complex operations in drilling is the lowering and raising of the column of drill pipes, which must be done regularly to replace the drill bit when it becomes dull. We note that in deep and superdeep drilling, the lowering and raising operations take up a large fraction of the total time and consist of the periodically repeated lowering (raising) of the column of drill pipes by a length of one drill-pipe stand (about 12-36 m). After this, the following stand is attached (disconnected), and the next lowering (raising) is carried out. These operations lead to the formation, in the drilling mud, of strong, periodically repeated disturbances, in the channel of the borehole, which, after propagating along the channel, and being reflected from its ends, superimposed, and damped, produce dynamic loads on the walls of the well, which often lead to different complications during drilling. Analogous effects arise during lowering of the column of the casings.

A number of theoretical studies devoted to this question are known [1, 2]. However, because of the complexity of the problem being considered, these studies completely or partially neglect such important factors as the unsteadiness of the phenomenon, the compressibility of the fluid, the non-Newtonian properties of the fluid, and the two-dimensionality of the flow picture.

We attempt to eliminate the indicated shortcomings.

We consider the following problem. We have a long vertical pipe of length L, radius R_2 with a closed end (Fig. 1), filled with a non-Newtonian compressible fluid with specific density ρ, having known rheology. Inside the pipe there is lowered another pipe, coaxial with it, having length $L_1 < L$, radius R_1 with an end that is closed by means of a valve, through which we can pump a fluid in a single direction with volume flow rate $q_1(t)$. The upper end of the annular pipe is open and communicates with the atmosphere.

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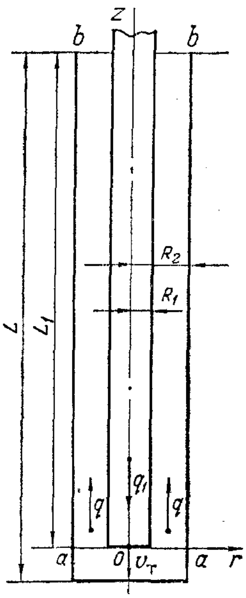

Fig. 1

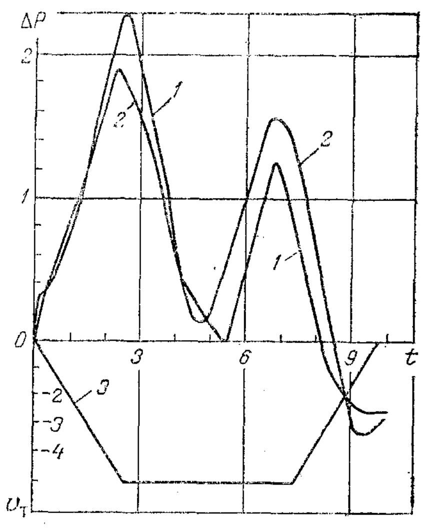

Fig. 2

Fig. 1. General view of the geometry of the channel.

Fig. 2. Variation with respect to time t (sec) of the hydrodynamic pressure component ΔP (MPa) in the lower section of the channel during motion of a column of inside pipes: 1) experimental curve; 2) calculated curve; 3) form of motion of inside pipe. v, m/sec.

At time t > 0, there begins the displacement of the inside column of pipes, consisting of three stages, during which the column accelerates, moves with constant velocity, and slows down to zero:

$$v_T(t)=\begin{cases} v_{T0}(t/\tau_1)^{k_1} & \text{for}\quad 0<t<\tau_1,\\ v_{T0} & \text{for}\quad \tau_1\leqslant t<\tau_1+\tau_2,\\ v_{T0}(1-[(t-\tau_1-\tau_2)/\tau_3]^{k_2}) & \text{for}\quad \tau_1+\tau_2<t\leqslant\tau_1+\tau_2+\tau_3,\\ 0 & \text{for}\quad t>\tau_1+\tau_2+\tau_3,\end{cases} \tag{1}$$

where $0<\{k_1,\ k_2\}\leqslant 10$.

Relationship (1) enables us to give a good approximation of the actual tachograms of the motion of the column of pipes, and subsequently to analyze the effect of the parameters τ_1, τ_2, τ_3, v_{T0}, k_1, and k_2 on the value of the hydrodynamic loads in the channel arising during its motion.

With motion of the inside pipe the fluid in the intertubular space in section (a—a) is displaced (Fig. 1) with volume flow rate $q_2(t) = \pi R_1^2 v_T(t)$. Because of the tangential friction of the wall of the inside pipe, a certain amount of viscous fluid is dragged along. Because the lower end of the outside pipe is closed, this flow of fluid is forced back into the intertubular space. Thus, the total volume flow rate through section (a—a) equals

$$q(t)=q_1(t)+q_2(t). \tag{2}$$

We assume that the fluid included between the closed end of the outside pipe and the section (a—a) is incompressible, and we neglect the effects connected with the motion of a fluid around the end face of the inside pipe with consideration of the flow in the intertubular space formed by the surfaces of the outside and inside pipes, the section (a—a), and the free surface of the fluid (b—b). We also neglect effects connected with the bulk viscosity and elasticity of the fluid, assuming that the characteristic frequencies of the disturbances and relaxations allow this.

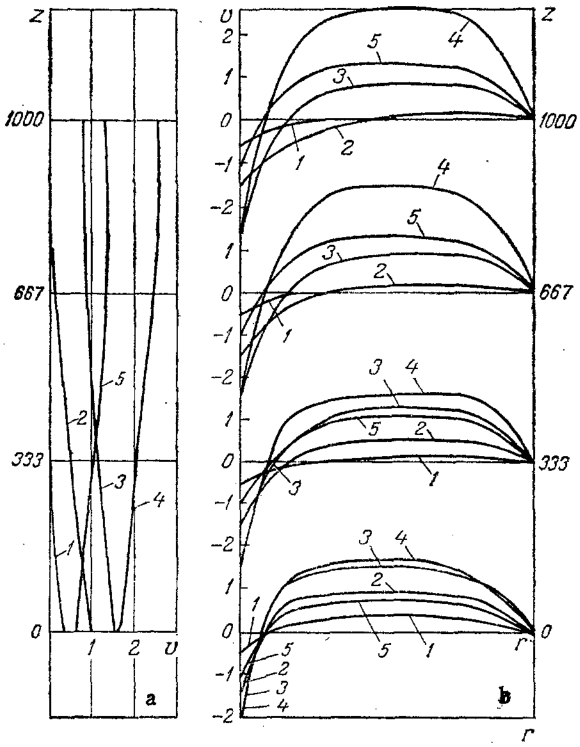

Fig. 3. Distribution of velocity v (m/sec): a) along channel at $r = R_1 + R_2/2$, $v = v(z, r = R_1 + R_2/2)$ at various times; b) form of velocity profiles at various channel sections ($v = v(r)$) for various times: 1) t = 0.8 sec; 2) 1.2; 3) 2.0; 4) 3.2; 5) 6.0 sec. z, m.

An expression for the most completely described unsteady motion of a compressible non-Newtonian fluid in the region being considered is the generalized Navier—Stokes equation [3, 4]:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \Delta \vec{v} = -\nabla P + \rho \vec{g} + \eta(S)\left(\Delta \vec{v} + \frac{1}{3}\operatorname{grad}\operatorname{div}\vec{v}\right) + 2\bar{D}\cdot\operatorname{grad}\eta(S) - \frac{2}{3}\operatorname{grad}\eta(S)\cdot\operatorname{div}\vec{v}, \tag{3}$$

where $\bar{D} = \frac{1}{2}(\nabla\vec{v} + \nabla\vec{v}^{\mathrm{T}})$ is the tensor of the rate of straining, and $S = 2\mathrm{tr}(\bar{D}^2)$ is twice the second principal invariant of this tensor.

As a rheological model of the fluid we take a power law [4]:

$$\eta(S) = kS^{(n-1)/2}. \tag{4}$$

To complete the system (3), (4), we use the equation of continuity over the section [5]:

$$-\frac{\partial P}{\partial t} = \rho c^2 \operatorname{div} u, \tag{5}$$

where $u = 2\int_{R_1}^{R_2} vrdr/(R_2^2 - R_1^2)$, i.e., for solution of the problem we make the assumptions that the pressure P is independent of the radius r, and the fluid is compressible only along the channel, but the fluid situated below the section (a—a) (length $L—L_1$) is incompressible. Such an approximation sufficiently well describes the actual picture of the flow for $L \gg (R_2 - R_1)$ and $L \gg (L - L_1)$.

We choose a cylindrical coordinate system (z, φ, r), the z axis of which coincides with the axis of the channel, and the origin of the coordinates is located in the lower face of the inside pipe (Fig. 1). Taking into account that the velocity of the fluid in the annular

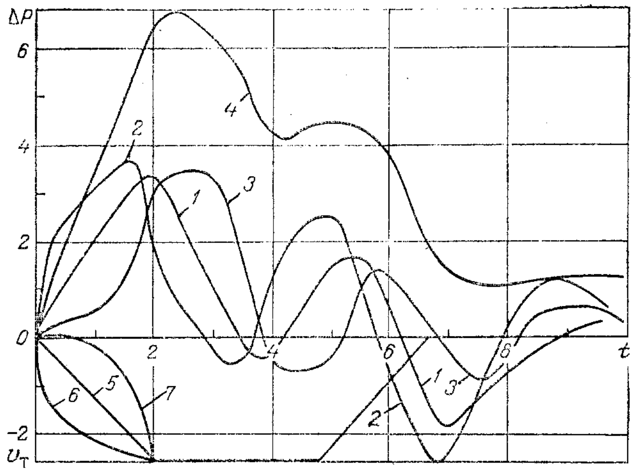

Fig. 4. Variation of hydrodynamic component of pressure ΔP (MPa) with respect to time t (sec): 1) with variation of velocity of inside pipe according to the law represented by curve 5; 2) with variation of velocity of inside pipe according to curve 6; 3) with variation of velocity of inside tube corresponding to curve 7; 4) with fourfold increase of the parameter k from the power-law rheological model and law of variation of the velocity of the inside pipe corresponding to curve 5.

channel in our formulation has only a z-component $v_z = v(r, z)$, we write Eqs. (3)-(5) in the chosen coordinate system [3]:

$$\rho\frac{\partial v}{\partial t}+\rho v\frac{\partial v}{\partial z}=-\frac{\partial P}{\partial z}+\eta\left(\frac{4}{3}\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)\rho g+\frac{4}{3}\frac{\partial v}{\partial z}\frac{\partial\eta}{\partial z}+\frac{\partial v}{\partial r}\frac{\partial\eta}{\partial z}, \qquad (6)$$

$$\frac{\partial P}{\partial t}=-c^2\rho\frac{\partial u}{\partial z},\quad \eta=k\left\{\left(\frac{\partial v}{\partial r}\right)^2+2\left(\frac{\partial v}{\partial z}\right)^2\right\}^{\frac{n-1}{2}},$$

$$\rho=\rho_0\left(1+\frac{P-P_0}{k}\right),\quad u=\frac{2\int\limits_{R_1}^{R_2} vrdr}{R_2^2-R_1^2}.$$

We assume that the channel is isothermal, and the flow is everywhere laminar. We assume the criterion for transition to turbulent flow to be the condition $Re' = \rho(D_2 - D_1)^n u^{2-n}/k\cdot 8^{n-1} > 2100$ [4], which shows that for real quantities, used in practice, the flow is mostly laminar in the annular channel.

As initial conditions we take the distribution of pressure and velocity that corresponds to hydrostatic conditions when there is no flow through the lower end of the inside pipe or steady flow with flow rate $q_1(t)$ in the opposite case. As the boundary condition in the upper section of the pipe we take the condition of the free surface $P = P_{atm}$, for which from the equation of continuity we find

$$\left.\frac{\partial v}{\partial z}\right|_{z=L_1}=0. \qquad (7)$$

In the lower section of the pipe (*a—a*) we are given the value of the mean velocity, obtainable from (2):

$$u=\frac{q(t)}{\pi(R_2^2-R_1^2)}. \qquad (8)$$

On the surfaces of the inside and outside pipes, the following adhesion condition is satisfied:

$$v(r = R_1) = v_T(t); \quad v(r = R_2) = 0. \tag{9}$$

The boundary-value problem (6), (7), (8), (9), formulated in this way, was solved numerically by the method of fractional intervals. The algorithm for the solution was discussed sufficiently thoroughly in [3].

A numerical experiment was carried out on the constructed model.

In Fig. 2 we compare the experimentally obtained time dependence of the pressure variation in the section $z \simeq L_1$ in the intertubular space of the well during lowering to 140 mm of drill pipes of length 1100 m, presented in [6], with the calculated curve obtained from a solution of the problem considered above.

The parameters of the fluid, given in [6], are: ρ = 1440 kg/m^3, T_0 = 11 Pa, η = 0.028 Pa·sec (the Bingham model of the fluid was approximately the same as the appropriate power model). The radii of the inside and outside pipes were R_1 = 0.07 m; R_2 = 0.146 m. The column of inside pipes was accelerated for τ_1 = 2.5 sec up to a velocity of 5.5 m/sec, through τ_2 = 4.8 sec there was damping for a time τ_3 = 2.5 sec.

As can be seen from Fig. 2, the calculations based on the assumed model give good agreement with experiment over the entire range of times being considered, which indicates that the selected model is adequate and the resulting effect is practical.

In Fig. 3 we present three-dimensional distributions of the velocities in the channel, obtained for solution of one of the variants being considered (ρ = 1650 kg/m^3, k = 11.2; n = 0.36, curve 5 in Fig. 4 corresponds to the chosen form of motion of the pipe) for various moments of time and coordinates. As follows from Fig. 3b, the profile of the flow in the channel always consists of two oppositely directed flows: a fluid dragged by tangential friction in the direction of motion of the inside pipe and a fluid moving in the opposite direction.

The velocity profiles in different sections (for z = const) differ from one another, since at all stages of the motion of the pipe, we have not succeeded in establishing processes for tracking the vibrational nature of the velocity variation over z. In the given figure we can observe a complicated picture of the interaction, including damping and reflection of the generated waves (Fig. 3a).

As follows from the solutions obtained, it is impossible to carry out estimates of the inertial forces, but this has been done in [1, 2]. Actually, the values of the velocities and accelerations differ at different points of the channel, whereas under the assumption of incompressibility of the fluid we come to an incorrect conclusion that the velocities at all sections of the channel at a given time should be the same. The estimates obtained in this way can be several times too large in comparison with the actual situation.

We finally note that the point of the profile at which the velocity v = 0 also oscillates according to the establishment of the flow in the channel. As was shown by calculations, this point has a tendency to shift to the right with increasing fluid viscosity and with approach to the outlet section of the pipe.

We consider one of the characteristic dimensions of the lowering of a pipe column of length L_1 = 1000 m, radius R_1 = 0.071 m in a well of radius R_2 = 0.114 m, filled with drilling mud of density ρ = 1650 kg/m^3 and rheology with parameters of the power-law model k = 2.8; n = 0.36. The flow rate through the inside pipe was assumed to be equal to q_1 = 0.0005 m^3/sec. The pipe column was accelerated according to the law (1) with various parameters. We set ourselves the problem of analyzing and drawing qualitative conclusions about the effect of various parameters of the fluid, geometry of the channel, and form of the motion of the inside pipe on the value of the hydrodynamic overloads arising at the walls of the channel for such motion.

In Fig. 4 we present the change in pressure below (near the section a—a) the channel (curves 1-4) as a function of the viscosity of the fluid and the nature of the motion of the column (curves 5-7). Here we consider cases when in expression (1) we have $\tau_1 = \tau_3$ = 2 sec; v_{T0} = —2.5 m/sec; τ_2 = 2.8 sec; k_2 = 1; k_1 = 0.25; 1; 4 (curves 6, 5, and 7 of Fig. 4). We note certain general relations obtained from the solution of the problems given in Figs. 3 and 4.

1. The picture of the pressure variation at any point of the channel has an oscillational character, with the period of the oscillations being equal to the total phase of the hydraulic shock.

2. The maximum value of the dynamic overload corresponds to the first maximum and occurs after completion of the acceleration stage of the pipe.

3. The value of this maximum depends strongly on the value of the velocity v_{T0}, the rheology and density, and the geometry of the channel (L_1, R_1, R_2), and it depends weakly on the acceleration of the pipe and its nature (k_1).

4. The oscillational damping time in the channel depends on the rheology of the fluid and occurs only for a certain time after the pipe has completely stopped.

5. Owing to reflections from the free surface and the arrival below the channel of unloading waves it is possible to have negative values of hydrodynamic loads, i.e., the pressure at any point of the channel can become less than the hydrostatic pressure (the first minimum on the curves 1-3). This phenomenon is confirmed by a complete series of experimental studies, e.g. [6].

6. The nature of the motion of the pipe on the first stage (k_1) affects the nature of the pressure increase in the channel and is always similar to it. In this case it affects not only the form of the leading edge of the pressure increase but also on the whole nature of its variation and, in particular, it leads to a certain phase shift of the curve (see Fig. 4, curves 1-3).

7. Deceleration of the inside column of pipes on the third stage of motion superimposes a new perturbation on the flow, which is still unsteady; as a rule, it leads to the appearance of negative overloads of pressure on the walls of the channel. Here, their value can reach several tens of atmospheres, and it depends not only on τ_3, v_{T0}, and k_2, but also on the moment at which deceleration begins with respect to the phase of the oscillations at a given point of the channel, caused by the first stage of motion of the inside pipe. Thus, if at a given moment, there is an increase in pressure at a definite point in the channel, then the deceleration of the inside pipe that has begun weakens it, and the negative value of the hydrodynamic overload will be a minimum (see Fig. 4, curve 3). If the moment of deceleration coincides with the decrease of the pressure wave, there will be a resonant amplification, and the value of the negative overload becomes a maximum (see Fig. 4, curve 2).

8. Thus, by selecting appropriate parameters for the motion of the pipe (k_1, k_2, τ_1, τ_2, τ_3, v_{T0}), we can minimize the value of the hydrodynamic loads on the walls of the channel.

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